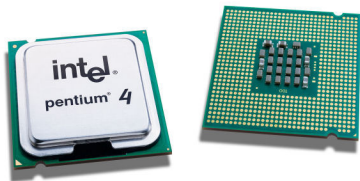


Why Distributed Algorithms?

Historical Context



So far, sequential algorithms
CPUs used to be single core,
not anymore!

Distributed (parallel) algorithms
make better use of hardware

Speedup

How much of a speedup can we actually get?

Depends on code itself

Amdahl's Law -

$$\frac{1}{1 - p + \frac{p}{s}}$$

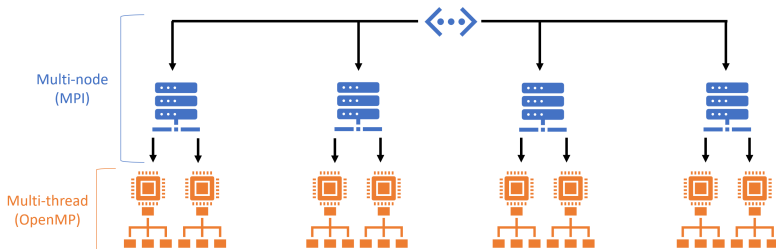
p - fraction of original execution time that can be optimized

s - speedup of optimized code over original code

This throws practical applications in the face of theory

distinction of $O(2n)$ and $O(n)$

Levels of Parallelism



Message Passing Interface (MPI)

Message Passing Interface (MPI)

MPI is a standard designed to allow for parallel functionality in a cluster

Every implementation has to follow this standard

Looks like function calls in code (library)

Pass data between processes as messages

- 1:1 transmit and receive



Hello World Example

MPI has a whole slew of new concepts

process - instance of the program being run

rank - unique identifier for a process (usually related to thread count)

communicators - groups together MPI processes

(Ex. `MPI_COMM_WORLD`)

finalization - clean up performed at end of MPI program

Does MPI Work?

Let's take a look at the example of bitonic sort.

Bitonic sorter

From Wikipedia, the free encyclopedia



This article **needs additional citations for verification**. Please help *improve this article* by **adding citations to reliable sources**. Unsourced material may be challenged and removed.

Find sources: "Bitonic sorter" – news · newspapers · books · scholar · JSTOR (October 2017) *(Learn how and when to remove this template message)*

Bitonic mergesort is a **parallel algorithm** for sorting. It is also used as a construction method for building a **sorting network**. The algorithm was devised by **Ken Batcher**. The resulting sorting networks consist of $O(n \log^2(n))$ comparators and have a delay of $O(\log^2(n))$, where n is the number of items to be sorted.^[1]

A sorted sequence is a monotonically non-decreasing (or non-increasing) sequence. A **bitonic** sequence is a sequence with $x_0 \leq \dots \leq x_k \geq \dots \geq x_{n-1}$ for some k , $0 \leq k < n$, or a circular shift of such a sequence.

Contents [hide]

- Complexity
- How the algorithm works
 - Alternative representation
- Example code
- See also
- References
- External links

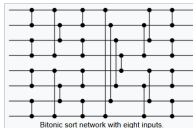
Complexity [edit]

Let $p = \lceil \log_2 n \rceil$ and $q = \lfloor \log_2 n \rfloor$.

It is obvious from the construction algorithm that the number of rounds of parallel comparisons is given by $q(q+1)/2$.

It follows that the number of comparators c is bounded $2^{p-1} \cdot p(p+1)/2 \leq c \leq \lfloor n/2 \rfloor \cdot q(q+1)/2$ (which establishes an exact value for c when n is a power of 2).

Bitonic sorter



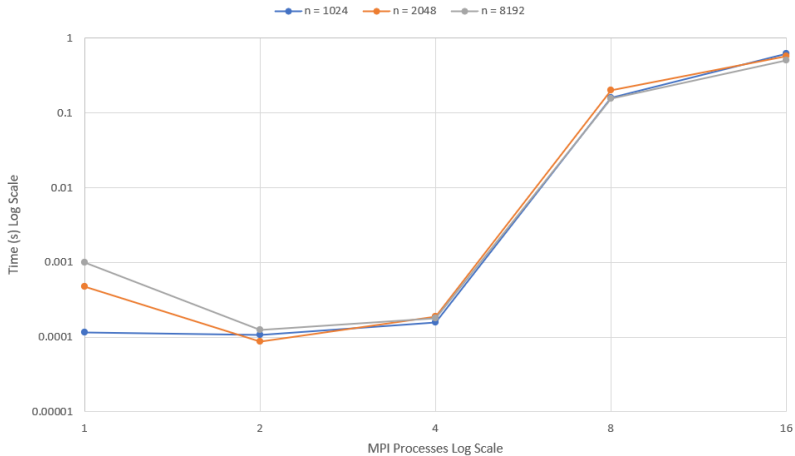
Class	Sorting algorithm
Data structure	Array
Worst-case performance	$O(\log^2(n))$ parallel time
Best-case performance	$O(\log^2(n))$ parallel time
Average performance	$O(\log^2(n))$ parallel time
Worst-case space complexity	$O(n \log^2(n))$ non-parallel time

Merge Sort Runtime: $O(n \log n)$

Parallel Bitonic Sort Runtime: $O(\log^2 n)$

Data for Bitonic Sort

Bitonic Sort Time (s) vs. MPI Processes
Intel i7-7500U @ 2.70 GHz 2c/4t



Calculating Pi Example

Mathematicians are very concerned (perhaps to a worrying degree) about finding all the digits of π .

To date, 62.8 trillion digits of π have been calculated.

Leibniz formula:

$$1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \dots = \frac{\pi}{4}$$

We can calculate this in a distributed fashion!