



LECTURE 5

ADVANCED GRAPH ALGORITHMS

Po Hao Chen

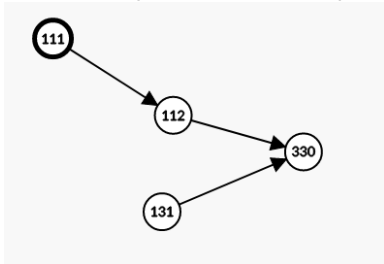
October 1, 2021

BOSTON
UNIVERSITY

Topological Sort

Recap

- If an event A has to occur before event B, we can model it as a Directed Acyclic Graph (DAG)
- Topological ordering is the valid sequence to process the DAG
 - For example: Software Dependencies, Prerequisites for Courses



111,112,131,330 or 131,111,112,330

Strongly Connected Components

Definition of SCCs

- a partition of a directed graph is a strongly connected component, if every vertex is reachable from every other vertex.

Routing Problem

- Given n cities and m routes, Can the capitol reach every single city?
If not how many minimum additional routes do we need to add?
- Caveat: Every road in this country is one-way.

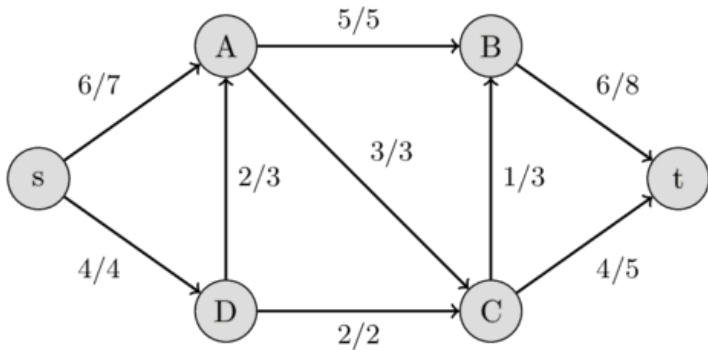
Kosaraju's Algorithm

- Original Graph + Transposed Graph
- Get Topological Order of Original Graph.
- Call DFS on Transposed Graph until all nodes are visited using the topological ordering. Each DFS call finds a unique strongly connected component.

Maxflow

Recap

- What is maxflow?



- Maxflow = Max. number of paths without shared edges
- Algorithm: Ford-Fulkerson, Edmond-Karp, Dinic

Edmond-Karp

Ford-Fulkerson Method:

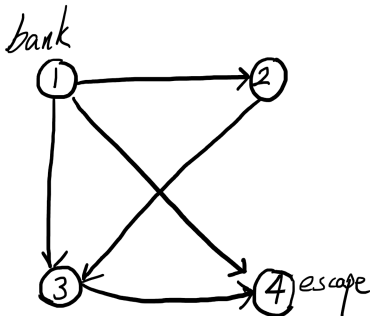
- Find augmenting path (simple path with no cycle)
- compute bottleneck capacity: i.e the minimum remaining capacity on the path
- augment each edge and total flow

Edmond-Karp is an improved implementation of Ford-Fulkerson.

- BFS to find the shortest augmenting path to sink.

Simulation Problem

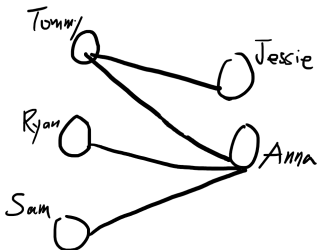
- A bank at location 1 has been robbed, the suspects are trying to get to location n to escape. What is the minimum number of streets the police need to close off to stop them?
- Output: minimum number of streets to close



Maximum-Bipartite Matching

Bipartite Matching Problem can be reduced to Max-Flow problem.

- You have been hired by a gossip girl, theorize who's going with who at prom based on their dating history.
- Output: max # of pairs, the pairs.



Min-Cost Flow

When **shortest path** meets **maximum flow**:

The MCMF algorithm is similar to Edmonds-Karp. Except, compute for lowest-cost path instead of lowest number of edges.

Armchair Problem

You want to make sure everyone moves from their original spot. it takes $|i - j|$ to go from i to j . What is the minimum time?

